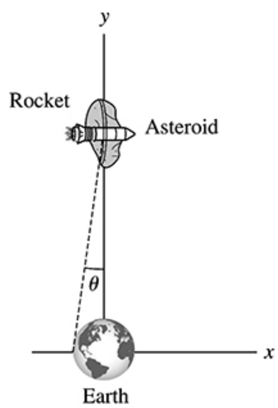
8.3. Model: The asteroid and the giant rocket will be treated as particles undergoing motion according to the constant-acceleration equations of kinematics.

Visualize:

Pictorial representation



Known

Rocket
$x_0 = 0 \quad y_0 = 4.0 \times 10^6$ km
$t_0 = 0 \quad v_{0y} = 0$
$F_x = 5.0 \times 10^9$ N

Asteroid
$x_0 = 0 \quad y_0 = 4.0 \times 10^6$ km
$t_0 = 0 \quad v_{0x} = 0$
$v_{0y} = 20$ km/s
$m = 4.0 \times 10^{10}$ kg
Radius of the earth = 6400 km

Find
$t_1 \quad \theta$

Solve: **(a)** The time it will take the asteroid to reach the earth is

$$\frac{\text{displacement}}{\text{velocity}} = \frac{4.0\times10^6 \text{ km}}{20 \text{ km/s}} = 2.0\times10^5 \text{ s} = 56 \text{ h}$$

(b) The angle of a line that just misses the earth is

$$\tan\theta = \frac{\text{R}}{y_0} \Rightarrow \theta = \tan^{-1}\left(\frac{\text{R}}{y_0}\right) = \tan^{-1}\left(\frac{6400 \text{ km}}{4.0\times10^6 \text{ km}}\right) = 0.092°$$

(c) When the rocket is fired, the horizontal acceleration of the asteroid is

$$a_x = \frac{5.0\times10^9 \text{ N}}{4.0\times10^{10} \text{ kg}} = 0.125 \text{ m/s}^2$$

(Note that the mass of the rocket is much smaller than the mass of the asteroid and can therefore be ignored completely.) The velocity of the asteroid after the rocket has been fired for 300 s is

$$v_x = v_{0x} + a_x(t - t_0) = 0 \text{ m/s} + (0.125 \text{ m/s}^2)(300 \text{ s} - 0 \text{ s}) = 37.5 \text{ m/s}$$

After 300 s, the vertical velocity is $v_y = 2\times10^4$ m/s and the horizontal velocity is $v_x = 37.5$ m/s. The deflection due to this horizontal velocity is

$$\tan\theta = \frac{v_x}{v_y} \Rightarrow \theta = \tan^{-1}\left(\frac{37.5 \text{ m/s}}{2\times10^4 \text{ m/s}}\right) = 0.107°$$

That is, the earth is saved.